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Contra *v*-Closed Mappings

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Abstract

The aim of this paper is to introduce and study the concept of Contra v-closed mappings and the interrelationship between other Contra-closed maps.

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Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-open mappings and in the year 2011 they further defined almost *v*-open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-closed and Almost *v*-closed mappings. Inspired with these concepts and its interesting properties we in this

paper tried to study a new variety of closed maps called contra *v*-closed maps. Throughout the paper X, Y means topological spaces (X, τ) and (Y,σ) on which no separation axioms are assured.

Preliminaries

Definition 1: $A\subseteq X$ is said to be

a) Regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A\subseteq cl(int(A))$; $A\subseteq int(cl(int(A)))$; $A\subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A))\subseteq A$; $cl(int(cl(A)))\subseteq A$; $int(cl(int(A)))\subseteq A$]

b) *v*-open if there exists regular-open set U such that $U\subseteq A\subseteq cl(U)$.

c) g-closed[rg-closed] if $cl(A)\subset U$[$rcl(A)\subset U$] whenever $A\subset U$ and U is open[r-open] in X and g-open[rg-open] if its complement X - A is g-closed[rg-closed].

Remark 1: We have the following implication diagrams for closed sets.

r-closed set → rα-closed set → *v*-closed set
↓ ↓
pre-closed set ← closed set → α-closed set → semi-closed set → β-closed set.

Definition 2: A function *f:X*→*Y* is said to be

a) continuous[resp:semi-continuous, r-continuous, *v*-continuous] if the inverse image of every open set is open [resp: semi open, regular open, *v*--open].
b) irresolute [resp: r-irresolute, *v*-irresolute] if the inverse image of every semi open [resp: regular open, *v*-open] set is semi open [resp: regular open, *v*-open].
c) closed[resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].
d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].
e) contra closed[resp: contra semi-closed; contra pre-closed; contra r*α*-closed] if the image of every closed set in *X* is open[resp: semi-open; pre-open; r*α*-open] in *Y*.

Definition 3: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

Contra *v*-Closed Mappings

Definition 1: A function *f: X*→*Y* is said to be contra *v*-closed if the image of every closed set in *X* is *v*-open in *Y*.
Theorem 1: Every contra *rα*-closed map is contra *v*-closed but not conversely.
Proof: Let A⊆*X* be closed ⇒ *f*(A) is *rα*-open in *Y* since *f:X*→*Y* is contra *rα*-closed
⇒*f*(A) is *v*-open in *Y* since every r*α*-open set is *v*-open. Hence *f* is contra *v*-closed.
Example 1: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {b, c}, *X*}; σ = {ϕ, {a}, {b}, {a, b}, *Y*}. Let *f:X*→*Y* be defined *f*(a) = b, *f*(b) = c and *f*(c) = a. Then *f* is contra *v*-closed, contra semi-closed, contra rα-closed and contra β-closed but not contra closed, contra pre-closed, contra r-closed, contra α-closed and contra rp-closed.
Example 2: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, {a, b}, *X*}; σ = {ϕ, {a, c}, *Y*}. Let *f:X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is contra pre-closed, contra rp-closed and contra β-closed but not contra closed, contra semi-closed, contra r-closed, contra *v*-closed, contra α-closed and contra rα-closed.
Theorem 2: Every contra *r*-closed map is contra *v*-closed but not conversely.
Proof: Let A⊆*X* be closed ⇒ *f*(A) is *r*-open in *Y* since *f:X*→*Y* is contra *r*-closed ⇒*f*(A) is *v*-open in *Y* since every r*α*-open set is *v*-open. Hence *f* is contra *v*-closed.
Example 3: Let *X* = *Y* = {a, b, c}; τ = {ϕ, {a}, *X*}; σ = {ϕ, {a}, {a, b}, *Y*}. Let *f:X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a. Then *f* is contra closed, contra semi-closed, contra pre-closed, contra β-closed, contra α-closed and contra rα-closed but not contra r-closed, contra *v*-closed but not contra rp-closed.
Theorem 3: Every contra *v*-closed map is contra semi-closed but not conversely.
Proof: Let A⊆*X* be closed ⇒ *f*(A) is *v*-open in *Y* since *f:X*→*Y* is contra *v*-closed ⇒*f*(A) is semi-open in *Y* since every *v*-open set is semi-open. Hence *f* is contra semi-closed.
Theorem 4: Every contra *v*-closed map is contra β-closed but not conversely.
Proof: Let A⊆*X* be closed ⇒ *f*(A) is *v*-open in *Y* since *f:X*→*Y* is contra *v*-closed ⇒*f*(A) is β-open in *Y* since every *v*-open set is β-open. Hence *f* is contra β-closed.
Note 1:

a) contra closed maps and contra *v*-closed maps are independent of each other.
b) contra *α*-closed map and contra *v*-closed map are independent of each other.
c) contra pre closed map and contra *v*-closed map are independent of each other.

Note 2: We have the following implication diagram among the open maps.

contra r-closed→ contra rα-closed → contra *v*-closed
↓ ↓
contra pre-closed←contra closed→ contra α-closed → contra-semi-closed→contra β-closed.
None is reversible.

Theorem 5: If R*α*O(*Y*) = *v*O(*Y*) then *f* is contra r*α*-closed iff *f* is contra *v*-closed.
Proof: Follows from theorem 3.1
Conversely Let A⊆*X* be closed ⇒*f*(A) is *v*-open in *Y* since *f:X*→*Y* is Contra *v*-closed ⇒ *f*(A) is *rα*-open in *Y* since every *v*-open set is *rα*-open. Hence *f* is Contra *rα*-closed.
Theorem 6: If *v*O(*Y*) = RO(*Y*) then *f* is Contra *r*-closed iff *f* is Contra *v*-closed.
Proof: Follows from theorem 3.2

Conversely Let A⊆X be closed ⇒f(A) is v-open in Y since f:X→Y is Contra v-closed ⇒ f(A) is r-open in Y since every v-open set is r-open. Hence f is Contra r-closed.

Theorem 7: If vO(Y) = αO(Y) then f is Contra α-closed iff f is Contra v-closed.

Proof: Let A⊆X be closed ⇒ f(A) is α-open in Y since f:X→Y is Contra α-closed ⇒f(A) is v-open in Y since every α-open set is v-open. Hence f is Contra v-closed.

Conversely Let A⊆X be closed ⇒f(A) is v-open in Y since f:X→Y is Contra v-closed ⇒ f(A) is α-open in Y since every v-open set is α-open. Hence f is Contra α-closed.

Theorem 8: If f is closed and g is Contra v-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed⇒ f(A) is closed in Y ⇒ g(f(A)) is v-open in Z ⇒ g o f(A) is v-open in Z. Hence g o f is Contra v-closed.

Theorem 9: If f is closed and g is Contra r-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed ⇒ f(A) is closed in Y ⇒ g(f(A)) is r-open in Z ⇒ g o f(A) is v-open in Z. Hence g o f is Contra v-closed.

Theorem 10: If f is closed and g is Contra $r\alpha$-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is closed in Y ⇒ g(f(A)) is $r\alpha$-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z Hence g• f is almost Contra v-closed.

Theorem 11: If f is r-closed and g is Contra v-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed ⇒ f(A) is r-closed in Y ⇒ g(f(A)) is v-open in Z ⇒ gof(A) is v-open in Z. Hence g o f is Contra v-closed.

Theorem 12: If f is r-closed and g is Contra r-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed ⇒ f(A) is r-closed in Y ⇒ g(f(A)) is r-open in Z ⇒ go f(A) is v-open in Z. Hence g o f is Contra v-closed.

Theorem 13: If f is r-closed and g is Contra $r\alpha$-closed then g o f is Contra v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is r-closed in Y ⇒ g(f(A)) is $r\alpha$-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z Hence g• f is Contra v-closed.

Corollary 1.1:

a) If f is closed[r-closed] and g is Contra v-closed then g o f is Contra semi-closed and hence Contra β-closed.
b) If f is closed[r-closed] and g is Contra r-closed then g o f is Contra semi-closed and hence Contra β-closed.
c) If f is closed[r-closed] and g is Contra $r\alpha$-closed then g o f is Contra semi-closed and hence Contra β-closed.

Theorem 14: If f is Contra closed and g is v-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is open in Y ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Theorem 15: If f is Contra closed and g is r-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is open in Y ⇒ g(f(A)) is r-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Theorem 16: If f is Contra closed and g is $r\alpha$-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is open in Y ⇒ g(f(A)) is $r\alpha$-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Theorem 17: If f is Contra-r-closed and g is v-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is r-open in Y ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Theorem 18: If f is Contra-r-closed and g is r-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is r-open in Y ⇒ g(f(A)) is r-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Theorem 19: If f is Contra-r-closed and g is $r\alpha$-open then g o f is Contra-v-closed.

Proof: Let A⊆X be closed in X ⇒ f(A) is r-open in Y ⇒ g(f(A)) is $r\alpha$-open in Z ⇒ g(f(A)) is v-open in Z ⇒ g•f(A) is v-open in Z. Hence g• f is Contra v-closed.

Corollary 1.2:

a) If f is Contra closed[Contra-r-closed] and g is v-open then g o f is Contra-semi-closed and hence Contra β-closed.
b) If f is Contra closed[Contra-r-closed] and g is r-open then g o f is Contra-semi-closed and hence Contra β-closed.

c) If f is Contra closed[Contra-r-closed] and g is $r\alpha$-clsoed then g o f is Contra-semi-closed and hence Contra β-closed.

Theorem 20: If f*:* $X\to Y$ is Contra v-closed, then $f(A^o)\subset v(f(A))^o$

Proof: Let A⊆ X be closed and f: X→ Y is Contra v-closed gives $f(A^o)$ is v-open in Y and $f(A^o)\subset f(A)$ which in turn gives $v(f(A^o))^o\subset v(f(A))^o$ - - - (1)

Since $f(A^o)$ is v-open in Y, $v(f(A^o))^o = f(A^o)$ - - - - - - - - - - - - - - - - - -(2)

combining (1) and (2) we have $f(A^o)\subset v(f(A))^o$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 1.3: If f:$X\to Y$ is Contra r-closed, then $f(A^o)\subset v(f(A))^o$

Proof: Let A⊆ X be closed and f: X→ Y is Contra r-closed gives $f(A^o)$ is r-open in Y and $f(A^o)\subset f(A)$ which in turn gives $v(f(A^o))^o\subset v(f(A))^o$ - - - - - - - - - -(1)

Since $f(A^o)$ is v-open in Y, $v(f(A^o))^o = f(A^o)$ - - - - - - - - - - - - - - - - - -(2)

combining (1) and (2) we have $f(A^o)\subset v(f(A))^o$ for every subset A of X.

Theorem 21: If f:$X\to Y$ is Contra v-closed and A⊆X is closed, f(A) is τ_{v}-open in Y.

Proof: Let A⊂ X be closed and f: X→ Y is Contra v-closed ⇒ $f(A^o)\subset v(f(A))^o \Rightarrow f(A) \subset v(f(A))^o$, since $f(A) = f(A^o)$. But $v(f(A))^o\subset f(A)$. Combining we get $f(A) = v(f(A))^o$. Therefore f(A) is τ_{v}-open in Y.

Corollary1.4: If f:$X\to Y$ is Contra r-closed, then f(A) is τ_{v}-open in Y if A is r-closed set in X.

Proof: Let A⊂ X be r-closed and f: X→ Y is Contra r-closed ⇒ $f(A^o)\subset r(f(A))^o \Rightarrow f(A^o)\subset v(f(A))^o$ (by theorem 3.20) ⇒ $f(A) \subset v(f(A))^o$, since $f(A) = f(A^o)$. But $v(f(A))^o\subset f(A)$. Combining we get $f(A) = v(f(A))^o$. Hence f(A) is τ_{v}-open in Y.

Theorem 22: If $v(A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:

a) f: X→ Y is Contra v-closed map

b) $f(A^o)\subset v(f(A))^o$

Proof: (a) ⇒(b) follows from theorem 3.20.

(b) ⇒ (a) Let A be any r-closed set in X, then $f(A) = f(A^o)\subset v(f(A))^o$ by hypothesis. We have $f(A) \subset v(f(A))^o$. Combining we get $f(A) = v(f(A))^o = r(f(A))^o$[by given condition] which implies f(A) is r-open and hence v-open. Thus f is Contra v-closed.

Theorem 23: f:$X\to Y$ is Contra v-closed iff for each subset S of Y and each open set U containing $f^{-1}(S)$, there is an v-closed set V of Y such that S⊆ V and $f^{1}(V) \subseteq U$.

Remark 3: Composition of two Contra v-closed maps is not Contra v-closed in general.

Theorem 24: Let X, Y, Z be topological spaces and every v-open set is closed[r-closed] in Y. Then the composition of two Contra v-closed[Contra r-closed] maps is Contra v-closed.

Proof: (a) Let f:$X\to Y$ and g:$Y\to Z$ be Contra v-closed maps. Let A be any closed set in X ⇒ f (A) is v-open in Y⇒ f (A) is closed in Y (by assumption) ⇒ $g(f$ (A)) is v-open in Z ⇒ g o f(A) is v-open in Z. Therefore g o f is Contra v-closed.

(b) Let f:$X\to Y$ and g:$Y\to Z$ be Contra v-closed maps. Let A be any closed set in X ⇒ f(A) is r-open in Y ⇒ f(A) is v-open in Y ⇒ f(A) is r-closed in Y (by assumption) ⇒ f(A) is closed in Y (by assumption) ⇒ $g(f$(A)) is r-open in Z ⇒ gof(A) is v-open in Z. Therefore gof is Contra v-closed.

Theorem 25: Let X, Y, Z be topological spaces and Y is discrete topological space in Y. Then the composition of two Contra v-closed[Contra r-closed] maps is Contra v-closed.

Theorem 26: If f:$X\to Y$ is g-closed, g:$Y\to Z$ is Contra v-closed [Contra r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then g o f is Contra v-closed.

Proof: (a) Let A be a closed set in X. Then f(A) is g-closed set in Y ⇒ f (A) is closed in Y as Y is $T_{1/2}$ ⇒ $g(f$ (A)) is v-open in Z since g is Contra v-closed ⇒ g o f (A) is v-open in Z. Hence g o f is Contra v-closed.

(b) Let A be a closed set in X. Then f(A) is g-closed set in Y ⇒ f (A) is closed in Y as Y is $T_{1/2}$ ⇒ $g(f$ (A)) is r-open in Z since g is Contra r-closed ⇒ g o f (A) is v-open in Z. Hence gof is Contra v-closed.

Corollary 1.5: If f:$X\to Y$ is g-open, g:$Y\to Z$ is Contra v-closed [Contra r-closed] and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is Contra p-closed and hence Contra β-closed.

Theorem 27: If f:$X\to Y$ is rg-open, g:$Y\to Z$ is Contra v-closed [Contra r-closed] and Y is r-$T_{1/2}$, then g o f is Contra v-closed.

Proof: Let A be a closed set in X. Then f(A) is rg-closed in Y ⇒ f(A) is r-closed in Y since Y is r-$T_{1/2}$ ⇒ f(A) is closed in Y since every r-closed set is closed ⇒ g(f(A)) is v-open in Z ⇒ g o f(A) is v-open in Z. Hence gof is Contra v-closed.

Corollary 1.6: If $f:X\to Y$ is rg-open, $g:Y\to Z$ is Contra v-closed [Contra r-closed] and Y is r-$T_{1/2}$, then $g\ o\ f$ is Contra semi-closed and hence Contra β-closed.

Theorem 28: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that gof is Contra v-closed [Contra r-closed] then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is Contra v-closed.
b) If f is g-continuous, surjective and X is $T_{1/2}$ then g is Contra v-closed.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is Contra v-closed.

Proof: (a) Let A be a closed set in $Y \Rightarrow f^{-1}(A)$ is closed in $X \Rightarrow (g\ o\ f)(f^{-1}(A))$ is v-open in $Z \Rightarrow g(A)$ is v-open in Z. Hence g is Contra v-closed.

(b) Let A be a closed set in Y. $\Rightarrow f^{-1}(A)$ is g-closed in $X \Rightarrow f^{-1}(A)$ is closed in X[since X is $T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A))$ is v-open in $Z \Rightarrow g(A)$ is v-open in Z. Hence g is Contra v-closed.

(c) Let A be a closed set in $Y \Rightarrow f^{-1}(A)$ is g-closed in $X \Rightarrow f^{-1}(A)$ is closed in X[since X is r-$T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}(A))$ is v-open in $Z \Rightarrow g(A)$ is v-open in Z. Hence g is Contra v-closed.

Corollary 1.7: If $f:X\to Y$, $g:Y\to Z$ be two mappings such that gof is Contra v-closed [Contra r-closed] then the following statements are true.

a) If f is continuous [r-continuous] and surjective then g is Contra semi-closed and hence Contra β-closed.
b) If f is g continuous, surjective and X is $T_{1/2}$ then g is Contra semi-closed and hence Contra β-closed.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is Contra semi-closed and hence Contra β-closed.

Theorem 29: If $f:X\to Y$ is Contra v-closed and A is an closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra v-closed.

Proof: (a) Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is v-open in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-closed.

Theorem 30: If $f:X\to Y$ is Contra r-closed and A is an closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra v-closed.

Proof: Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is r-open in Y. $\Rightarrow f(A)$ is v-open in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-closed.

Corollary 1.8: If $f:X\to Y$ is Contra v-closed [Contra r-closed] and A is an closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra semi-closed and hence Contra β-closed.

Theorem 31: If $f:X\to Y$ is Contra v-closed, X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra v-closed.

Proof: Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is v-open in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-closed.

Theorem 32: If $f:X\to Y$ is Contra-r-closed, X is $T_{1/2}$ and A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra v-closed.

Proof: Let F be a closed set in A. Then F = A∩E for some closed set E of X and so F is closed in $X \Rightarrow f(A)$ is r-open in $Y \Rightarrow f(A)$ is v-open in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-closed.

Corollary 1.9: If $f:X\to Y$ is Contra v-closed [Contra r-closed], X is $T_{1/2}$, A is g-closed set of X then $f_A:(X, \tau(A)) \to (Y, \sigma)$ is Contra semi-closed and hence Contra β-closed.

Theorem 33: If $f_i: X_i\to Y_i$ be Contra v-closed [Contra r-closed] for i =1, 2. Let $f: X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$. Then $f:X_1 x X_2\to Y_1 x Y_2$ is Contra v-closed.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is closed in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is v-open set in $Y_1 x Y_2$. Then $f(U_1 x U_2)$ is v-open set in $Y_1 x Y_2$. Hence f is Contra v-closed.

Corollary 1.10: If $f_i: X_i\to Y_i$ be Contra v-closed [Contra r-closed] for i =1,2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is Contra semi-closed and hence Contra β-closed.

Theorem 34: Let $h:X\to X_1 x X_2$ be Contra v-closed. Let $f_i:X\to X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then $f_i: X\to X_i$ is Contra v-closed for i =1, 2.

Proof: Let U_1 be closed in X_1, then $U_1 x\ X_2$ is closed in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is v-open in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore f_1 is Contra v-closed. Similarly we can show that f_2 is also Contra v-closed and thus f_i: X $\to X_i$ is Contra v-closed for i = 1, 2.

Corollary 1.11: Let $h: X\to X_1 x X_2$ be Contra v-closed. Let $f_i: X\to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then $f_i: X\to X_i$ is Contra semi-closed and hence Contra β-closed for i =1,2.

Conclusion

In this paper we introduced the concept of contra *v*-closed mappings, studied their basic properties and the interrelationship between other closed maps.

References

[1] Asit Kumar Sen and Bhattcharya.P., On preclosed mappings, Bull.Cal.Math.Soc.,85, 409-412(1993).

[2] Baker.C.W., Contra-open functions and contra-closed functions, Math. Today (Ahmedabad) 15 (1997), 19–24.

[3] Balasubramanian.S., and Vyjayanthi.P.A.S., *v*-open Mappings – Scientia Magna Vol 6. No. 4(2010) 118 – 124.

[4] Balasubramanian.S., and Vyjayanthi.P.A.S., *v*-closed Mappings – Jr. Advanced Research in Pure Mathematics, Vol 3. No. 1(2011)135 – 143

[5] Balasubramanian.S., *v*g-open mappings – Inter. J. Comp. Math. Sci. and Application Vol.5. No.2(2011)7-14.

[6] Balasubramanian.S., and Krishnamurthy.T.K., Regular pre-Closed mappings – Inter. J. Math. Archive, Vol 2, No. 8(2011) 1411 – 1415.

[7] Balasubramanian.S., Vyjayanthi.P.A.S., and Sandhya.C., Almost *v*-closed Mappings – Inter. J. Math. Archive,Vol 2, No. 10 (2011) 1920 – 1925.

[8] Balasubramanian.S., Sandhya.C., and Vyjayanthi.P.A.S., Almost *v*-open Mappings – Inter. J. Math. Archive,Vol 2, No.10 (2011) 1943 – 1948.

[9] Balasubramanian.S., and Lakshmi Sarada.M., *gpr*-closed and *gpr*-open functions- International Journal of Mathematical Engineering and Science, Vol.1,No.6(2012)09 – 16.

[10] Balasubramanian.S., and Lakshmi Sarada.M., Almost *gpr*-closed and Almost *gpr*-open functions- Aryabhatta Journal of Mathematics and Informatics (In press)

[11] Caldas,M., and Baker,C.W., Contra Pre-semiopen Maps, Kyungpook Math.Journal, 40 (2000), 379 – 389.

[12] Di Maio. G., and Noiri.T, I. J. P. A. M., 18(3) (1987) 226-233.

[13] Dontchev.J., Mem.Fac.Sci.Kochi Univ.ser.A., Math., 16(1995), 35-48.

[14] Dunham.W., $T_{1/2}$ Spaces, Kyungpook Math. J.17(1977), 161-169.

[15] Long.P.E., and Herington.L.L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.

[16] Malghan.S.R., Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82 -88.

[17] Mashour.A.S., Hasanein.I.A., and El.Deep.S.N., α-continuous and α-open mappings, Acta Math. Hungar.,41(1983), 213-218.

[18] Noiri.T., A generalization of closed mappings, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54 (1973),412-415.

[19] Noiri.T., Almost αg-closed functions and separation axioms, Acta Math. Hungar. 82(3) (1999),193-205.

[20] Palaniappan.N., Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).

[21] Vadivel.A., and Vairamanickam.K., *rg*α-Closed and *rg*α-open maps in topological spaces. Int.Journal of Math. Analysis, 2010, 4(10): 453 - 468.